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FUNDAMENTAL NATURAL FREQUENCY FOR ISOTOPIC RECTANGULAR PLATE SIMPLY SUPPORTED ON THREE EDGES WITH ONE EDGE FREE OF SUPPORT (SSSF PLATE)

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ABSTRACT

The paper presents a theoretical formulation based on polynomial shape function and application of Ritz method. In this study, the free vibration of simply supported panel with one free edge was analyzed. The Polynomial shape function derived was substituted into the potential energy functional, which was minimized to obtain the fundamental natural frequency. This research focused on aspect ratio from 0.1 to 2.0 with 0.1 increments. The values of fundamental natural frequencies of the first mode obtained were compared with those of previous research works. The present values for aspect ratio 0.5, 1.0 and 1.3 were 4.9469, 12.3435 and 19.1582 respectively.

KEYWORDS: Fundamental Natural Frequency, SSSF Plate, Ritz Method, Polynomial Shape Function

1. INTRODUCTION

There has not been any exact solution on the vibration of rectangular panels with one free edge. Over the years, problems have been treated by the use of trigonometric series as the shape function of the deformed panel or by using method of superposition. Research has been carried out on the problems from equilibrium approach and others solved the problems from energy and numerical approaches. However, no matter the approach used, trigonometric series has been the most widely used shape functions. The analysis of the free vibration of panels was well documented by Leissa (1973) and includes a variety of boundary conditions and aspect ratio using trigonometric series. Gorman (1982) solved problem on free vibration of rectangular panel with different boundary conditions, aspect ratio and Poisson ratios using method of superposition.

2. FORMULATION OF FUNDAMENTAL NATURAL FREQUENCY

Chakraverty (2009) gave the maximum kinetic energy functional as

$$K_{max} = \frac{\lambda^2}{2}\iint \rho h W^2(x,y)\partial x \partial y \quad (1)$$

Making use of the non dimensional parameters, R and Q, equation (1) becomes

$$= \frac{ab\lambda^2 \rho h}{2}\iint w^2 \partial R \partial Q \quad (2)$$

Where ρ is the weight per unit area of the plate, h is the plate thickness and λ is the frequency of panel vibration. The maximum strain energy functional for a thin rectangular isotropic panel under vibration was given by Ibearugbulem (2012) as follow:

$$\text{Umax} = \frac{D}{2}\iint [(W''^{x})^2 + 2(W''^{xy})^2 + (W''^{y})^2]\partial x\partial y \quad (3)$$

Adding equations (2) and (3) gave the total potential energy functional of rectangular panel under lateral vibration as:

$$\prod max = \frac{aDb}{2}\iint \left[\frac{1}{a^4}(W''^{R})^2 + \frac{2}{a^2b^2}(W''^{RQ})^2 + \frac{1}{b^4}(W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (4)$$

Factorizing out b/a^3 gave:

$$\prod max = \frac{Db}{2a^3}\iint \left[(W''^{R})^2 + \frac{2a^2}{b^2}(W''^{RQ})^2 + \frac{a^4}{b^4}(W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (5)$$

If P= a/b, then:

$$\prod max = \frac{Db}{2a^3}\iint [(W''^{R})^2 + 2P^2(W''^{RQ})^2 + P^4(W''^{Q})^2]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (6)$$

If p= b/a, then:

$$\prod max = \frac{Db}{2a^3}\iint \left[(W''^{R})^2 + \frac{2}{P^2}(W''^{RQ})^2 + \frac{1}{P^4}(W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (7)$$

From equation (4), if a/b^3 is factorized out then:

$$\prod max = \frac{Da}{2b^3}\iint \left[\frac{b^4}{a^4}(W''^{R})^2 + \frac{2b^2}{a^2}(W''^{RQ})^2 + (W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (8)$$

If P = a/b, then:

$$\prod max = \frac{Da}{2b^3}\iint \left[\frac{1}{P^4}(W''^{R})^2 + \frac{2}{P^2}(W''^{RQ})^2 + (W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (9)$$

If P = b/a, then:

$$\prod max = \frac{Da}{2b^3}\iint [P^4(W''^{R})^2 + 2P^2(W''^{RQ})^2 + (W''^{Q})^2]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (10)$$

TAYLOR-MCLAURIN'S SERIES SHAPE FUNCTION

Displacement Function for SSFS and SSSF Panel

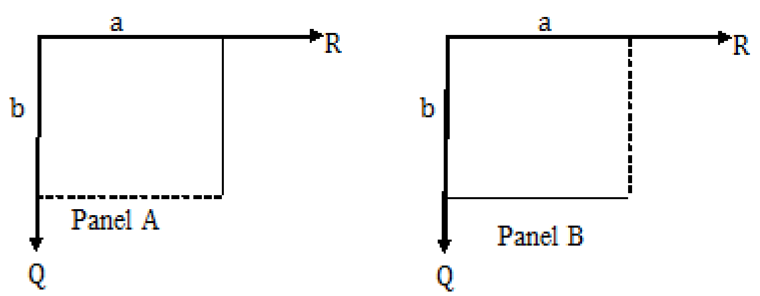

Figure 1

Deflection Equation

The deflection equation, according to Ibearugbulem (2012), is:

$$W = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4)(b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \quad (11)$$

Meaning that $W = W^R . W^Q$ (12)

Boundary Condition (for panel A)

$W(Q=0)=0, W(R=0)=0$

$W''(Q=0)=0, W''(R=0)=0$

$VR(Q=1)=0, W(R=1)=0$

Boundary condition (for panel B)

$W(R=0)=0, W(Q=0)=0$

$W''(R=0)=0, W''(Q=0)=0$

$VR(R=1)=0, W''(Q=1)=0$

For simply supported edges, both kinematic and dynamic boundary conditions were satisfied. At the free edge, the kinematics boundary conditions were satisfied. The dynamic boundary conditions are the shear force and bending moment. Only the shear force condition was satisfied at the free edge. Using panel B, these gave:

$W^R = a_4 (8R-4R^3+R^4)$ (13)

For W^Q

$W^Q = b_4 (Q-2Q^3+Q^4)$ (14)

Remembering from that

$W = W^R . W^Q$

$W = A (8R-4R^3+R^4) (Q-2Q^3+Q^4)$ (15)

The above equation is the displacement function for SSSF panel while the displacement equation/function for

SSFS was $W = A (R-2R^3+R^4) (8Q-4Q^3+Q^4)$ (16)

Total Potential Energy for SSSF Panel

Integrating the squares of the differential equation (15) with respect to R and Q

$$\int_0^1\int_0^1 (W''^R)^2 \partial R \partial Q = A^2(76.8)(0.04920634921) = 3.779047619\, A^2$$

$$\int_0^1\int_0^1 (W'^{R'Q})^2 \partial R \partial Q = A^2(31.08571429)(0.485714857) = 15.09877551 A^2$$

$$\int_0^1\int_0^1 (W''^Q)^2 \partial R \partial Q = A^2(12.5968254)(4.8) = 60.46476192 A^2$$

$$\int_0^1\int_0^1 (W)^2 \partial R \partial Q = A^2(12.5968254)(0.04920634921) = 0.618437896 A^2$$

Substituting the results into integral (6), (7), (9) and (10)

If P = a/b

$$\prod max = \frac{DbA^2}{2a^3}[3.779047619 + 30.19755102P^2 + 60.46476192P^4] - \frac{ab\lambda^2\rho hA^2}{2}[0.6198437896] \quad (17)$$

P= b/a

$$\prod max = \frac{DbA^2}{2a^3}\left[3.779047619 + \frac{30.19755102}{P^2} + \frac{60.46476192}{P^4}\right] - \frac{ab\lambda^2\rho hA^2}{2}[0.6198437896] \quad (18)$$

If P = a/b

$$\prod max = \frac{DaA^2}{2b^3}\left[\frac{3.779047619}{P^4} + \frac{30.19755102}{P^2} + 60.46476192\right] - \frac{ab\lambda^2\rho hA^2}{2}[0.6198437896] \quad (19)$$

If P=b/a

$$\prod max = \frac{DaA^2}{2b^3}[3.779047619P^4 + 30.19755102P^2 + 60.46476192] - \frac{ab\lambda^2\rho hA^2}{2}[0.6198437896] \quad (20)$$

Minimizing (17), (18), (19), (20) by $\frac{\partial \prod max}{\partial A} = 0$ and making λ subject of formula. Table 1, 2, 3 and 4 gives the results of fundamental natural frequency and the comparison of present result with other research works.

Figure 1 to 8 shows the graphical model of parameter (k) of fundamental natural frequency results, for both X-X and Y-Y axis

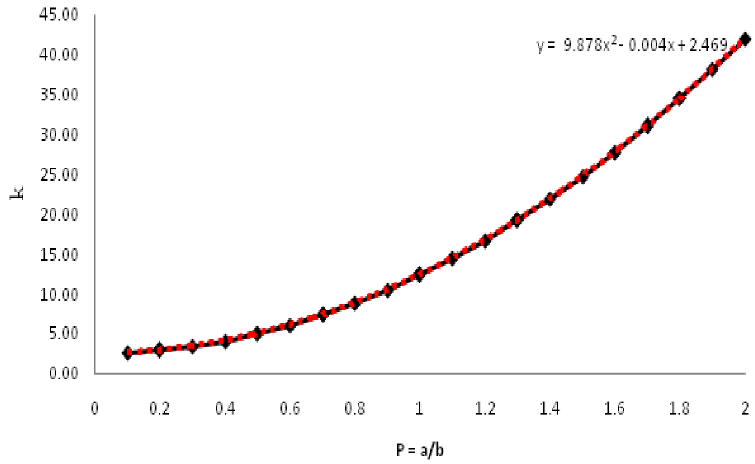

Figure 1: Graph of SSSF Panel, P=a/b (Note: y = k and x = Aspect Ratio, p)

The free edge of the panel is on X – X axis. With respect to length (a), the natural frequency increases as the aspect ratio increases, the polynomial equation curve is represented by y =9.878x^2 - 0.004x + 2.469.

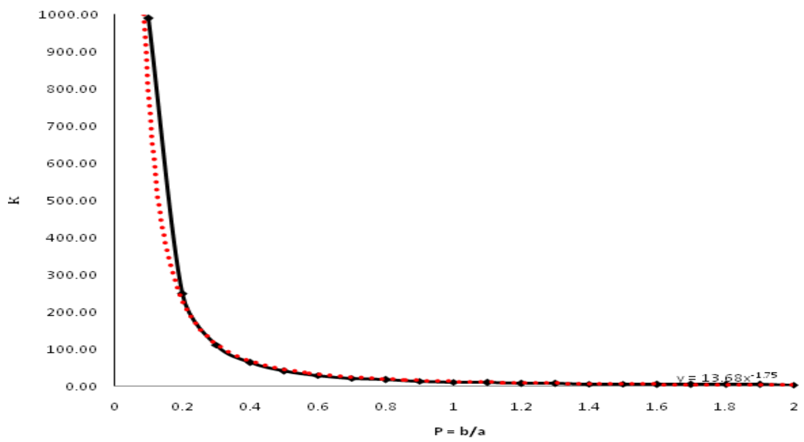

Figure 2: Graph of SSSF Panel, P=b/a (Note: y = k and x = Aspect Ratio, p)

The free edge of the panel is on X – X axis. With respect to length (a), the natural frequency decreases as the aspect ratio increases, the power equation curve is represented by $y = 13.68x^{-1.75}$

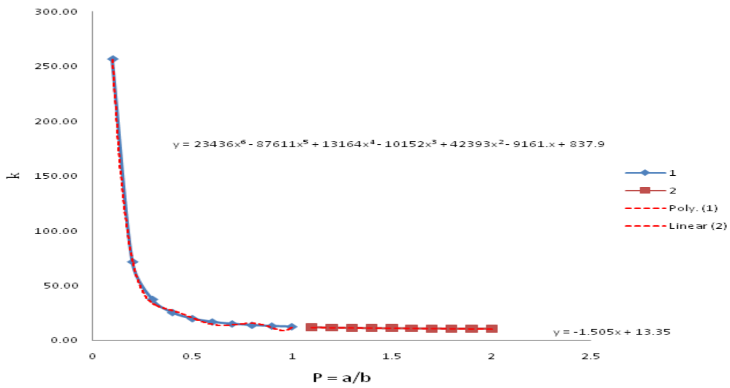

Figure 3: Graph of SSSF Panel, P = a/b (Note: Y = k and x = Aspect Ratio, p)

The free edge of the panel is on X – X axis. With respect to width (b), the natural frequency decreases as the aspect ratio increases. This model consist of one polynomial equation $y = 23436x^6 - 87611x^5 + 13164x^4 - 10152x^3 + 42393x^2 - 9161.x + 837.9$ which stop at P= 1 and linear equation $y = -1.505x + 13.35$.

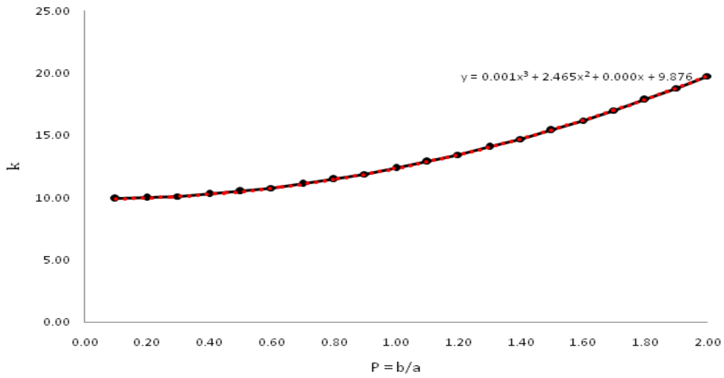

Figure 4: Graph of SSSF Panel, P = b/a (Note: y = k and x = Aspect Ratio, p)

The free edge of the panel is on X – X axis. With respect to width (b), the natural frequency increases as the aspect ratio increases, the polynomial equation curve is represented by $y =0.001x^3 + 2.465x^2 + 0.000x + 9.876$.

SSFS PANEL

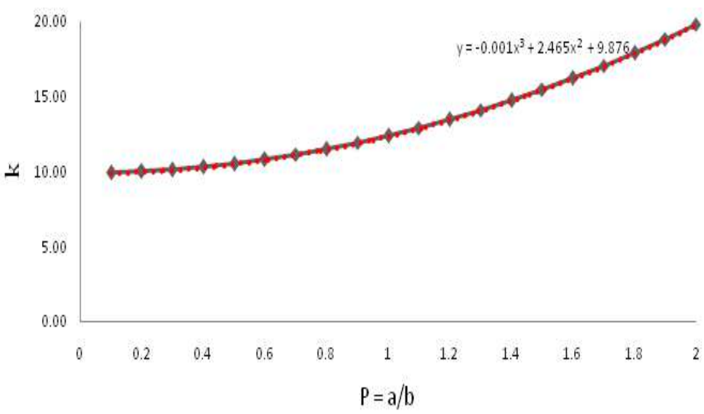

Figure 5: Graph of SSFS, P = a/b (Note: y = k and x = Aspect Ratio, p)

The free edge of the panel is on Y – Y axis. With respect to length (a), the natural frequency increases as the aspect ratio increases, the polynomial equation curve is represented by $y = 0.001x^3 + 2.465x^2 + 0.000x + 9.876$.

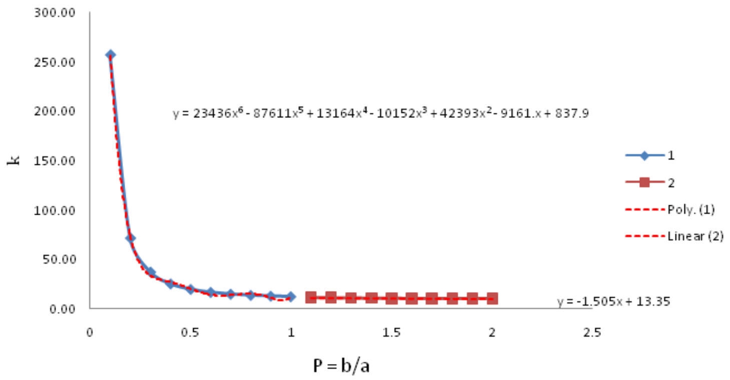

Figure 6: Graph of SSFS, P = b/a (Note: y = k and x = Aspect Ratio, p)

The free edge of the panel is on X – X axis. With respect to length (a), the natural frequency decreases as the aspect ratio increases. This model consist of one polynomial equation $y = 23436x^6 - 87611x^5 + 13164x^4 - 10152x^3 + 42393x^2 - 9161.x + 837.9$ which stop at P= 1 and linear equation $y = -1.505x + 13.35$.

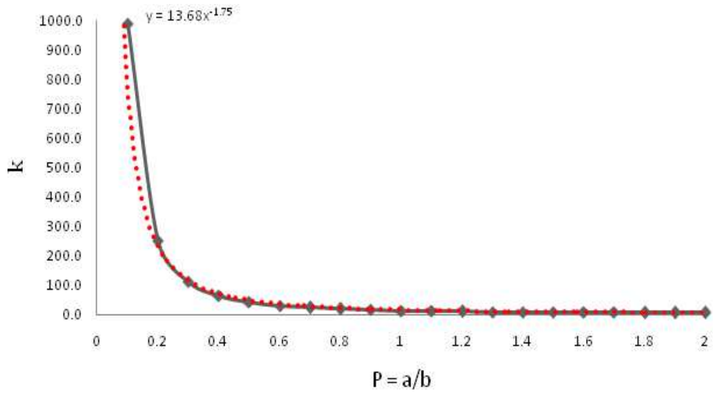

Figure 7: Graph of SSFS, P = a/b (Note: y = k and x = Aspect Ratio, p)

The free edge of the panel is on Y-Y axis. With respect to width (b), the natural frequency decreases as the aspect ratio increases, the power equation curve is represented by $y = 13.68x^{-1.75}$

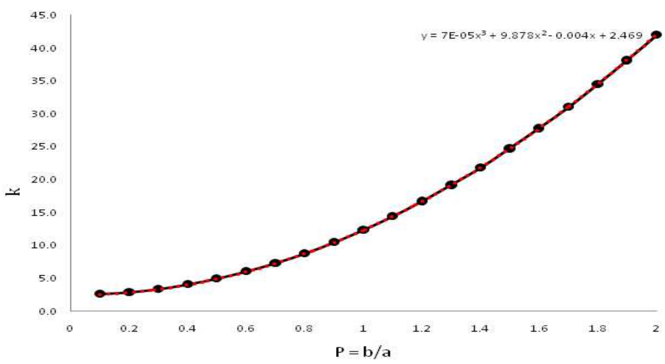

Figure 8: Graph of SSFS, P = b/a, (Note: y = k and x = Aspect Ratio, p)

The free edge of the panel is on Y-Y axis. With respect to width(b), the natural frequency increases as the aspect ratio increases, the polynomial equation curve is represented by $y = 7E\text{-}05x^3 + 9.878x^2 - 0.004x + 2.469$.

COMPARISON OF RESULTS FOR SSFS

The table compared the results of SSFS from previous researchers with difference in percentage. In table 1, the present result was compared the results of SSFS from Lessia (1973) and the present result ranges from 1.44% to 12.48% given an average difference of 5.11% which is ok. Table 2 and 3 displayed the difference between Gorman (1982) and present result with aspect ratio P =a/b and P =b/a respectively. The average difference is/are 0.0167% and -7.04% respectively.

Table 1: SSFS, P = b/a

$\lambda = \frac{K}{a^2}\sqrt{\frac{D}{\rho h}}$			
$P = \frac{a}{b}$	Present	Lessia (1973)	% Diff
0.4	10.2713	10.1259	1.44
0.6	10.7646	10.6712	0.88
1.0	12.3436	11.6845	5.64
1.5	15.4218	13.7111	12.48

Table 2: SSFS, P = b/a

$\lambda = \frac{K}{a^2}\sqrt{\frac{D}{\rho h}}$			
$P = \frac{b}{a}$	Present	Gorman (1982)	% Diff
1.0	12.3436	12.56	-1.75
1.5	10.9729	10.90	0.64
2.0	10.4933	10.37	1.16

Table 3: SSFS, P = a/b

$\lambda = \frac{K}{b^2}\sqrt{\frac{D}{\rho h}}$			
$P = \frac{a}{b}$	Present	Gorman (1982)	% Diff
1.0	9.3259	12.56	-1.75
1.5	6.8570	7.379	-7.033
2.0	4.9369	5.630	-12.33

Other researcher's worked on aspect ratio P = 1, as shown in table 4

Table 4: SSFS, P = 1

P = 1	1	2	3
λ	11.6898	11.3731	11.684

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